International Journal of Theoretical Physics, Vol. 44, No. 11, November 2005 (© 2005)
DOI: 10.1007/s10773-005-8977-z

Who Proved Haag's Theorem?

Tracy Lupher[1]

In physics literature, there are several different characterizations of Haag's theorem and its consequences for quantum field theory. These different versions of Haag's theorem are due in part to various generalizations and more "rigorous" proofs of Haag's theorem as well as to the fact that many of these proofs were done using different formulations of quantum field theory. As a result, there is confusion about what Haag's theorem is and when it was proved. This paper clears up some of these confusions by examining the history and development of Haag's theorem up to 1959. It is argued that the question of who proved Haag's theorem is tied up with what the theorem is taken to show.

KEY WORDS: Haag's theorem; van Hove; Friedrichs; canonical commutation relations; unitarily inequivalent representations; 11.10.-z; 11.10.Cd.

1. INTRODUCTION

Haag's original theorem assumed that there are two sets of field operators that satisfy the canonical commutation relations: (1) the free, or asymptotic fields, which occur at time $t = \pm\infty$, and (2) the "actual" fields, which occur at any finite time and that the theory can be formulated entirely from them. He also assumed that (3) there is a unique invariant normalizable vacuum state for the theory, that (4) there is a positive definite energy spectrum, and that (5) the "actual" fields are transformed by unitary operators representing translations in space. From these assumptions, Haag showed that (1) and (2) cannot belong to the same representation of the canonical commutation relations; they are unitarily inequivalent representations.

Haag's theorem is an important result in quantum field theory that has not been examined in much depth in the philosophy of physics literature. What it is and what follows from it has been an area of controversy in physics. Philosophers of physics have tended to view it as an important area for study, but few have done much to clarify the nature and scope of Haag's theorem. To the extent that philosophers of physics do discuss Haag's theorem, their analysis is promissory at best.

[1] Department of Philosophy, The University of Texas at Austin; e-mail: lupher.t@mail.utexas.edu

2. WHAT IS HAAG'S THEOREM?

Haag's theorem is generally taken to show that there are severe to insurmountable mathematical difficulties modeling interactions in quantum field theory. The nature and extent of these mathematical difficulties are the loci of debate about the significance (or insignificance) of Haag's theorem. Here is how Teller, a philosopher of physics, described Haag's theorem:

> According to something called Haag's theorem, there appears to be no known consistent mathematical formalism within which interacting quantum field theory can be expressed. (Teller, 1995)

A more conservative description is given by the physicists Streater and Wightman.

> Haag's theorem is very inconvenient; it means that the interaction picture exists only if there is no interaction (Streater and Wightman, 2000).

Streater and Wightman do not make the stronger statement, as Teller did. Rather, they take Haag's theorem to show that the interaction picture is empty of interactions in canonical quantum field theory on Fock space.[2] But in either case, Haag's theorem is a significant result for the foundations of quantum field theory. What conclusions have philosophers of physics reached on Haag's theorem?

3. CONFLICTING REACTIONS OF PHILOSOPHERS OF PHYSICS

As shocking as Haag's theorem appears, philosophers of physics have done very little to explicate it. The few who mention it tend to regard it as something important that someone (else) should investigate thoroughly.

> [Haag's theorem] implies, for example, that the only QFTs that exist in the interaction picture describe free fields. Since this is the framework used by physicists to describe the interacting theories of nature, the theorem seemingly presents a paradox (Huggett and Weingard, 1996).

> Everyone must agree that as a piece of mathematics, Haag's theorem is a valid result that at least appears to call into question the mathematical foundations of interacting quantum field theory, and agree that at the same time the theory has proved astonishingly successful in application to experimental results. What seems less clear is how the assumptions of the theorem should be brought to bear on both the product and the interpretation of the theory . . . I have no light to throw on these important questions. In this chapter, my exposition will proceed along lines almost universally accepted by practitioners of the theory, disregarding Haag's theorem. (Teller, 1995)

[2] Presumably, one reason they did not make the stronger statement put forth by Teller was the limited success of constructive field theory; the demonstrated ability to model specific interactions in two- and three-dimensions but not yet in four-dimensions.

But there is also a dissenting view in the philosophy of physics literature about the significance of Haag's theorem.

> There may be a presence within a theory of conceptual problems that appear to be the result of mathematical artifacts. These seem to the theoretician to be not fundamental problems rooted in some deep physical mistake in the theory, but, rather, the consequence of some misfortune in the way in which the theory has been expressed. Haag's Theorem is, perhaps, a difficulty of this kind. (Sklar, 2000)

One thing to notice in these quotations is the use of modifiers such as "seemingly," "at least appears to," and "perhaps" in their discussion of Haag's theorem. No one has taken a very firm stand on what the consequences of Haag's theorem may be. Nor has anyone provided an argument for the significance or insignificance.[3] A plausible explanation for this hesitancy to take a stand on Haag's theorem is obtained by doing a quick search of the physics literature. One will find many different papers claiming to prove, "rigorously" prove, or to prove a "generalized" version of Haag's theorem. These proofs are sometimes done under different formulations of quantum field theory such as Wightman's axiomatic approach or the LSZ approach. Thus, it is not clear whether Haag's theorem applies generally or only to some approaches to quantum field theory and not to others.

4. CONFLICTING REACTIONS OF PHYSICISTS

There are several different opinions about the significance of Haag's theorem in the physicist community. For example, Wightman and Roman considered Haag's theorem an important result in quantum field theory.

> [T]here is a widespread opinion that the phenomena associated with Haag's Theorem are somehow pathological and irrelevant for real physics ... I make one more attempt to explain why that is not the case. (Wightman, 1965)

> Haag's Theorem is very deep. . .The most sobering consequence of Haag's theorem is that the interaction picture of canonical field theory cannot exist unless there are no interactions. (Roman, 1969)

On the insignificance side of the debate is Källén who said the following:

> [T]he theorem discovered by van Hove and Friedrichs and usually referred to as the "Haag theorem" is really of a very trivial nature and it does not mean that the eigenvalues of a Hamiltonian never exist or anything that fundamental. (Källén, 1962)

The connection between Haag's theorem and certain problems with the Hamiltonian that Källén mentioned in the quotation above will be discussed below in connection with van Hove's work.

[3] The only exception that I have found is an article by Heathcote (1989), but his article is mainly focused on his view of causality.

Other field theorists choose not to worry about Haag's theorem or its possible implications on their work. Their calculations have been empirically verified, and they have little concern about a mathematical result that says that they may not be calculating the results of various interactions.

> [L]et us first ask what we are to make of it when we find practicising field theorists plunging ahead, presenting their theory with blithe indifference to the problems posed by Haag's theorem. As I understand the history of the subject, quantum field theory was developed in ignorance of these mathematical problems. Indeed, the theory was initially formulated and applied with astonishing empirical success in the late 1940s, while the difficulties here in question did not come to light until the mid 1950s. Even the existence of the problems did not become widely known. And even when they were appreciated, most field theorists were not about to let such formal problems get them sidetracked from the obviously impressive successes of their theories. Work continued as if these formal problems did not exist—theorists took a "Damn the mathematical torpedoes, full speed ahead!" attitude (Teller, 1998).

Some physicists who have heard of Haag's theorem misunderstand it. Teller (1998) recalled one instance of talking to a "prominent field theorist" about Haag's theorem who incorrectly dismissed it as having to do with issues of mathematical rigor associated with the delta function. It is stories like these that most likely led to Streater (1975) calling Haag's theorem, "one of the most widely misquoted results of the subject [quantum field theory]."[4]

5. THE NEGLECT OF HAAG'S PROOF

If one looks for a discussion of Haag's theorem in books on quantum field theory, one will hardly find a mention of it in textbooks written after the 1970s. The standard textbooks of quantum field theory such as Peskin and Schroeder (1995), Ryder (1996), and Weinberg (1995) do not mention Haag's theorem. If one wants to find textbook discussions of Haag's theorem, then it is necessary to look at textbooks written in the 1960s and 1970s, e.g. Roman (1969), Barton (1963), Streater and Wightman (2000; originally published in 1964), and Bogolubov *et al*. (1975). However, these discussions of Haag's theorem focus on Hall and Wightman's proof of a "generalized" Haag's theorem and they do so in the context of Wightman's axiomatic formulation of quantum field theory. There is rarely any analysis of Haag's original proof. This begs the question as to why these quantum field theorists do not bother to analyze Haag's original proof.

[4] Streater cited as an example of this the textbook by Bjorken and Drell (1965). In their chapter on perturbation theory, they assumed that the interacting and incoming free fields were connected at each time t by a unitary transformation. In a footnote on p. 175, they stated that the existence of such a unitary transformation breaks down for systems with a nondenumerable number of degrees of freedom and cite Haag's 1955 paper, but then they assume the existence of such a unitary transformation!

Part of the answer is that in the 1960s and 1970s, quite a bit of work was being done on new approaches to quantum field theory, while Haag's original paper was based on quantum field theory from the late 1940s through the early 1950s. I have only found two sources that do more than merely cite Haag's paper. In Hall and Wightman's paper in which they proved their generalized Haag's theorem, they indicated that Haag's original proof was inconclusive.

> In the opinion of the present authors, Haag's proof is, at least in part, inconclusive . . . . It will not escape the discerning reader of Haag's paper that, while we have generalized his results, eliminated one of his assumptions (the asymptotic condition), completed his proofs, and sharpened his conclusions, the essential physical points are Haag's. (Hall and Wightman, 1957)

Unfortunately, they do not characterize the shortcomings of Haag's proof or why it was inconclusive. The other source is a review of Haag's paper by Dyson (1955), where he said that the "so-called Haag's Theorem . . . is essentially an old theorem of L. Van Hove but is here presented in much greater generality." Dyson also took Haag to task somewhat for not providing any constructive solution to the problem of interactions. Though it is unclear why Dyson is so critical of Haag's paper, there is one reason to be suspicious. In the abstract of Haag's paper, he wrote, "It is shown that the "free field vacuum" of the Tamm-Dancoff method and Dyson's matrix $U(t_1, t_2)$ for finite t_1 or t_2 cannot exist" (Haag, 1955).

6. INFLUENCES ON HAAG'S FORMULATION OF THE THEOREM

Dyson and Källén suggest (in the quotations above) that they felt that Haag's theorem was based on the work of van Hove and Friedrichs. From Haag's original paper (1955), we know that he was familiar with the work of van Hove (1952) and Friedrich (1953) as well as a preprint of the paper by Wightman and Schweber (1955). We also know that the ideas for Haag's paper were presented in lectures that he gave at the CERN theoretical study group from 1952–1953. I will give a brief review of these influences in this section.

6.1. Van Hove's Work

There are two main papers of van Hove (1951, 1952) that are usually cited in connection with Haag's theorem and the mathematical problems involved in modeling interactions in quantum field theory. In his 1951 paper, van Hove investigated the mathematical properties of the interaction Hamiltonian H_{I} and the total Hamiltonian $H = H_{\mathrm{B}} + H_{\mathrm{F}} + gH_{\mathrm{I}}$, where H_{B} is the free boson Hamiltonian, H_{F} is the free fermion Hamiltonian, H_{I} is the interaction Hamiltonian, and g is a dimensionless coupling constant, in quantum field theory.[5] He was interested in

[5] If $g = 0$, there is no interaction and only free fields are present.

whether H and H_{I} exist as well-defined operators on the Hilbert space S_o of the normalized stationary states φ_α of the free fields. To answer this question, van Hove assumed that the system is put in a cubic box with periodic boundary conditions on the walls, which was a typical assumption for the time (Wentzel, 1949). The periodic boundary conditions change the continuous space into a lattice. Within each box, the momentum of the total and interaction Hamiltonian is cut off for some value K and the stationary states φ_α have finite energy and are characterized by having a finite number of particles (bosons, fermions, and anti-fermions) in specific plane-wave states. The original Hamiltonians are recovered formally by $H = \lim_{K\to\infty} H^K$ and $H_I = \lim_{K\to\infty} H_I^K$. S_o is the Hilbert space formed from linear combinations of the φ_α vectors: $\varphi = \Sigma_\alpha c_\alpha \varphi_\alpha$ with $\varphi_\alpha \in S_o$, c_α complex, and $\Sigma_\alpha |c_\alpha|^2 < \infty$. S_o is the domain of the free Hamiltonians for fermions and bosons and H^K and H_I^K are densely defined operators on it. The main result of this paper is that for any non-zero vector $\varphi = \Sigma_\alpha c_\alpha \varphi_\alpha$ in S_o, the total Hamiltonian and the interaction Hamiltonian cannot be defined on S_o because they have infinite norms: $H\varphi = \Sigma_\alpha c'_\alpha \varphi_\alpha$ and $H_I\varphi = \Sigma_\alpha c''_\alpha \varphi_\alpha$ with $\Sigma_\alpha |c'_\alpha|^2 = +\infty$ and $\Sigma_\alpha |c''_\alpha|^2 = +\infty$. Even if $H^K\varphi$ and $H_I^K\varphi$ are vectors in S_o, when the cutoff is removed by taking the limit as K goes to infinity the resulting operator yields a vector that is not in the Hilbert space of the free fields: $\lim_{K\to\infty} \|H^K\varphi\| = \infty = \lim_{K\to\infty} \|H_I^K\varphi\|$.[6]

While van Hove is generally credited as the first person to demonstrate some of the mathematical problems with treating the Hamiltonian as a well defined operator in quantum field theory, there are two sources that he cited where there problems were discussed earlier. Snyder (1950) mentioned that the Hamiltonian when applied to a state vector maps that state vector into a vector of infinite length (1950). Van Hove obtained some preliminary results for his 1951 paper collaboration with Gossiaux whose dissertation (1950) was on the domain of the Hamiltonian in quantum field theory.

In both his 1951 and 1952 papers, van Hove believed that infinite tensor product spaces might be the appropriate mathematical structure to model interactions and on which the Hamiltonian is well defined. In his 1951 paper, he wanted to expand the space of stationary states to include not only states where there are a finite number of particles present, but also stationary states where there are an infinite number of particles present. There are a nondenumerable number of these states and they have infinite eigenvalues from $H_{\mathrm{B}} + H_{\mathrm{F}}$. This much larger Hilbert space contains S_o as a subspace. Van Hove conjectured that H could be defined and diagonalized in this larger space using vectors that spanned the S_g subspaces, which depend on the value of the coupling constant g. For different values of g, including the free field case of $g = 0$, the subspaces S_g and $S_{g'}$ are *orthogonal*

[6] As van Hove pointed out, this implies that any normalized superposition of stationary states of the free field will have infinite average values for the square of the total and interaction Hamiltonians (when $g \neq 0$): $\langle H^2\rangle_\phi = \|H\phi\|^2 = +\infty$ and $\langle H_I^2\rangle_\phi = \|H_I\phi\|^2 = +\infty$.

($g \neq g'$). Van Hove's result that the space of free states is orthogonal to the space of interacting states is often cited with reference to his 1952 paper, but he had already anticipated this result in his 1951 paper.

In his 1952 paper, van Hove examined the case of a neutral-scalar field that was in scalar interaction with infinitely heavy fixed point sources. In this case, an exact solution can be obtained and compared with the perturbative solution. Van Hove stated that the origin of the divergences in this case was due to the fact that the space of stationary states for the free field is orthogonal to the state space of the stationary states of the field interacting with the sources. The implications of this are nicely summarized in Coleman's (1953) review of van Hove's 1952 paper. "[Van Hove's result] suggests that there is no mathematical justification for using the interaction representation and the occasional successes of renormalization methods are lucky flukes." Van Hove showed that while the exact solution and the method of renormalized perturbations give the same S-matrix, they disagree on the unitary matrix $U(-\infty, t)$ for finite t. Since the original Dyson framework for quantum field theory relies on doing series expansions using such unitary matrices van Hove's result showed that these (unrenormalized) matrices do not exist. The nonexistence of the $U(-\infty, t)$ matrix was one of the results that Haag claimed to show in the quotation above.

6.2. Friedrichs' Work

While van Hove investigated ultraviolet divergences ($\vec{\mathbf{k}} = \infty$), Friedrichs investigated infrared divergences ($\vec{\mathbf{k}} = 0$) of bosons interacting with a source distribution in his 1953 book, *Mathematical Aspects of Quantum Field Theory*. He defined creation and annihilation operators in terms of the field operator and the field's canonical conjugate momentum operator. He then showed that there are representations of the creation and annihilation operators which satisfy their canonical commutation relations but for which the number operator is not defined. He called a representation of the field and its canonical conjugate momentum *amyriotic* if the total number operator is well defined and he called it *myriotic* if the total number operator is not well defined. The most striking feature of myriotic fields is that they do not possess a vacuum state, i.e., the no-particle state. In the case of infrared divergences, Friedrichs showed that the representation of the field operators is myriotic. This accounted for the problems associated with defining the Hamiltonian. Friedrichs showed that if one used myriotic fields then the Hamiltonian could be defined. This is roughly similar to van Hove's suggestion that the Hamiltonian could be defined if one allowed states that contained an infinite number of particles since myriotic fields do not have states that a finite number of particles. Friedrichs also showed that in certain cases the unitary operator $U(t)$

does not have a limit as $t \to \infty$, which again showed that there were problems constructing a unitary operator $U(t, \infty)$ for finite t.

6.3. Wightman and Schweber's Paper

There was bidirectional influence between Haag on the one hand and Wightman and Schweber on the other. Haag was given a preprint of the Wightman and Schweber's paper by Wightman when he was working on his 1955 paper, and Wightman and Schweber had access to some unpublished CERN lectures of Haag given in 1953. The Wightman and Schweber paper also highlighted the difficulties of making the Hamiltonian a well-defined operator. They showed that if a certain condition is satisfied, then the Hamiltonian of a system is well-defined for just one value of the coupling constant. On the basis of the work of van Hove, they showed that if a certain condition is satisfied while the uncoupled or free fields has a vacuum state, the equations of motion may show that the coupled system may not have a vacuum state. This undermined perturbation theory which assumes that both the free field and the coupled system have a vacuum state. This also showed that representations which require the existence of a vacuum state may be inconsistent with the equations of motion.

7. HAAG'S PROOF IN HIS 1955 PAPER

The work cited above was known to Haag when he wrote his 1955 paper. The result in that paper which can be classified as "Haag's theorem" is that the field operators corresponding to the asymptotic, or free fields, which occur at time $t = \pm\infty$, and the field operators corresponding to the "actual fields," which occur at some finite time, belong to unitarily inequivalent representations of the canonical commutation relations (CCRs). His proof is a *reductio ad absurdum*. Suppose the free fields and the "actual fields" *were* unitarily equivalent and satisfy the CCRs. Haag then showed two things: (1) The vacuum states of the two representations would have to be the same vacuum state. (2) It follows from (1) that the free fields must satisfy a different set of canonical commutation relations. Thus, the "actual fields" and the free fields belong to unitarily inequivalent representations of the CCRs.

The connection with van Hove's result is the following. Haag showed that the representations of the "actual" and free fields as operators acting on Hilbert spaces cannot be unitarily equivalent. Haag also assumed that these representations are irreducible. In modern operator theory, it is a well-known mathematical fact that two irreducible representations are unitarily inequivalent if and only if they are orthogonal. Thus, van Hove's result that the Hilbert spaces of the stationary state space of the free fields is orthogonal to the stationary state space of the interacting field ($g \neq 0$) is contained within Haag's theorem because, as Haag noted (1955), he does not use a particular form of the Hamiltonian in his proof. In this sense, Haag generalized van Hove's result and it would be more appropriate to call Haag's theorem a generalized van Hove theorem because the essential physical points come from van Hove's work.

8. THE HALL-WIGHTMAN-GREENBERG PROOF OF HAAG'S THEOREM 1957–59

As was mentioned earlier, Hall and Wightman did not find Haag's proof conclusive. In their 1957 paper, they proved a generalized Haag's Theorem that came in two parts. (1) Given two neutral scalar fields that are related at a finite time by a unitary transformation, that satisfy the CCRs, and that have unique normalizable vacuum states that are invariant under Euclidean transformations (translations and rotations); it then follows that the unitary transformation takes the vacuum state of the first-field theory to the vacuum state of the second-field theory multiplied by a constant whose absolute value is equal to one. (2) If two field theories satisfy the assumptions of (1) and they and their vacuum states are invariant under inhomogeneous Lorentz transformations and have no negative energy states, then the first four vacuum expectation values are equal for all times. This showed that no interaction could be modeled by the usual representation of the creation and annihilation operators where the first four vacuum expectation values differed from the free-field values. As Hall and Wightman pointed out, their result is only valid up to the first four vacuum expectation values. Thus, even the Hall and Wightman result did not completely prove Haag's theorem. This was accomplished by Greenberg (1959), who used mathematical induction to prove that the first n vacuum expectation values are equal (for any positive integer n).

9. SUMMARY

Determining who proved "Haag's theorem" depends on one's particular understanding of what the theorem is about. Since the Hamiltonian is the generator of time translations, van Hove's work, which showed that the interaction Hamiltonian and the total Hamiltonian are not well defined on the space of stationary states of the free field, indicates that there would be problems constructing a unitary operator that connects the free fields to the interacting fields. Haag's proof can be seen as a generalization of van Hove's result, which shows only in a particular case that the stationary state space of the free field is orthogonal to the stationary state space of the field interacting with sources. Haag provided the first steps towards the more modern way of discussing the problem in terms of the unitary (in)equivalence of representations of the CCRs, In this way, Haag's paper united the work of van Hove with Friedrichs' work on the representations of the CCRs. If Haag's theorem is roughly understood to be a result that shows that an interacting field theory will have the same expectation values as a free-field theory (one would expect them to be different), then the Hall-Wightman-Greenberg papers are a proof of that idea. However, if Haag's theorem is taken to be about how the interacting fields (or the interaction Hamiltonian) cannot be defined using the same canonical commutation relations (or Hilbert space, respectively) as that of free fields, then the work of van Hove, Friedrichs, Wightman, Schweber, and Haag proved this in many cases. Though Haag's original 1955 proof and the "generalized" version of Haag's theorem in Hall and Wightman (1957) have some similarities, they have significant differences. Haag wanted to show that the free-field representation and the "actual" or interacting field representation are not unitarily equivalent representations of the CCRs. Hall and Wightman showed that under certain conditions, two field theories will, up to four vacuum expectation values, have the same values. The 1959, generalized version of the Hall-Wightman proof by Greenberg was used to show when a field theory will be equivalent to the free-field theory without concerning itself with how the free fields might be constructed through some asymptotic condition.

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